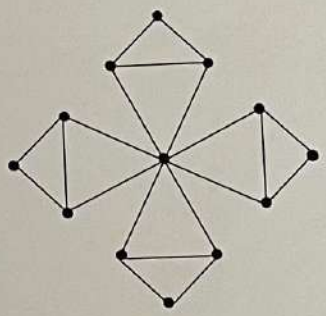
Exam Graph Theory

22 January 2024, 15.00–17.00

- It is absolutely not allowed to use calculators, phones, computers, books, notes, the help of others or any other aids.
- Always make sure to state clearly any results from the lecture notes you are using.
- Write the answer to each question on a separate sheet, **with your name and student number on each sheet**. This is worth 10 points (out of a total of 100).

Exercise 1 (20 pts).
Determine a maximum matching in the following graph, and provide a proof that it is a maximum matching.



(*Hint*: for the proof you may wish to use the Berge-Tutte formula. As mentioned above, make sure to clearly state any results from the lecture notes you are using.)

Exercise 2 (30 pts)
The following table gives the distances in km between 5 cities.

$i \setminus j$	1	2	3	4	5
1	0	51	31	27	88
2	51	0	42	77	37
3	31	42	0	56	61
4	27	77	56	0	11
5	88	37	61	11	0

Apply Dijkstra's algorithm to determine all shortest paths from vertex 1 to each other vertex.

Make sure to clearly indicate, for each step of the algorithm, what step the algorithm takes and the values of all the relevant variables.

Exercise 3 (20 pts)
For G a graph, let $\overline{G} := \left(V(G), \binom{V(G)}{2} \setminus E(G)\right)$ denote its *complement*.
Show that if $v(G) \geq 11$ then at most one of $G, \overline{G}$ is planar.
(*Hint*: how many edges can a planar graph have?)

Exercise 4 (20 pts).
Show that $\chi(G) \geq 3$ if and only if G contains a cycle of odd length.

(The end)